Modeling the SM Higgs Boson Mass Signal

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Abstract

A physically motivated model has been developed in order to characterize and extract a mass measurement for the SM Higgs boson in the Higgs to four leptons (H->4l) decay channel. A Breit-Wigner (BW) probability distribution function (pdf) convoluted with the Crystal Ball (CB) function was used for this purpose. The BW pdf was expected to characterize the physical aspects of the Higgs boson, while the CB pdf picks up various detector and final state radiation effects that would distort the observed shape of the mass signal. The mean and sigma of a Gaussian fit to the pull distribution taken from a high statistics "toy Monte Carlo" experiment for a simultaneous fit to the four decay products in the H->4l channel in the presence of background were found to be 0.047 ± 0.010 and 1.009 ± 0.007 respectively. These results show that the model is mostly unbiased with a good estimate on the uncertainty of the mass measurement. The estimated uncertainty on a mass measurement using this model was determined to be 0.6914 ± 0.0054 GeV.

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1. Introduction

The search for the standard model (SM) Higgs boson is one of the main objectives of the LHC at CERN. The Higgs mechanism, which provides a description as to how the massive elementary particles attain their masses, was the only particle predicted by the SM that had not yet been discovered. In July of this year both the ATLAS and CMS collaborations independently measured an excess of events at a mass of ~ 126 GeV with a local significance of 5.9 [1] and 5.0 [2] sigmas respectively. One of the many important necessities for observations like this to be made is a robust model that can accurately and precisely characterize the data in such a way that a mass measurement can be extracted. In this paper an attempt is made to develop a new, physically motivated model that could provide a better interpretation of the mass signal, and a more precise measurement of the Higgs mass.

Before describing how the model was built, a quick introduction is given to the ATLAS detector, production and decay of the Higgs boson, one of the main backgrounds present with the mass signal, and the different samples used for the analysis. The procedure for building the model is then described, followed by a discussion of the results of the analysis, and finally an outlook as to what may be done next to improve the model.

2. The ATLAS Detector

The ATLAS detector is a multipurpose particle-physics apparatus built specifically for the energy scales present at the LHC. The detector is centered on the LHC beam line with forward-backward cylindrical symmetry [3]. It can be subdivided into four main sections: the inner tracking detector (ID), the electromagnetic calorimeter (EC), the hadronic calorimeter, and the muon spectrometer. The ID itself is composed of a silicon pixel detector, a silicon microstrip detector, and a straw-tube transition radiation tracker (TRT) [3]. The EC is a high-granularity liquid Argon sampling calorimeter that is subdivided into a central barrel and identical end cap regions on either side of the detector. The entire detector is surrounded by a large muon spectrometer consisting of three large air-core superconducting magnets, a precision tracking system, and fast triggering detectors [3]. The combination of all these systems gives the detector its impressive size of 7000 tons, with a length of ~45 m, and radius of more than 25m.

3. Higgs Production, Decay, and Background

Gluon-gluon fusion (ggF) is the main production mechanism for a SM Higgs boson over the entire Higgs mass range investigated at the LHC [1]. To a good approximation, the ggF process proceeds through a top quark loop to the Higgs particle [4]. The ggF process was the only production mechanism that was analyzed in this paper.

The two main decay channels of the Higgs that contributed to the excess signal analyzed by the ATLAS experiment are the Higgs to diphoton decay, $H \to \gamma\gamma$, and the so called Higgs to four leptons decay, $H \to 4l$. The focus of this paper is on the second of these channels. In the $H \to 4l$ channel, the Higgs initially decays into a ZZ(*) boson pair who subsequently decay into the four leptons. The identity of the daughter leptons in this decay theoretically may be any of the leptons, however due to the capabilities of the ATLAS detector, only the decay into electron and muon daughters are described. Restricting ourselves to these possibilities produces a total of four different combinations within the $H \to 4l$ decay channel, corresponding to the decays $H \to \mu^+\mu^-\mu^+\mu^-$, $H \to \mu^+\mu^-e^+e^-$, $H \to e^+e^-e^+e^-$, and $H \to e^+e^-\mu^+\mu^-$. A distinction is made between the two channels $H \to \mu^+\mu^-e^+e^-$, and $H \to e^+e^-\mu^+\mu^-$ due to a difference in how the ATLAS event selection is completed. A diagram showing the entire Higgs production and decay process analyzed in this report is shown in Figure 3.1.

The H → 4l channel is a low background channel that is dominated by the irreducible four-lepton contribution from ZZ(*) production arising from quark-antiquark and gluon-gluon processes [1]. This background contribution produces a continuous spectrum across the whole mass range analyzed. The ZZ(*) line shape is shown in Figure 3.2.

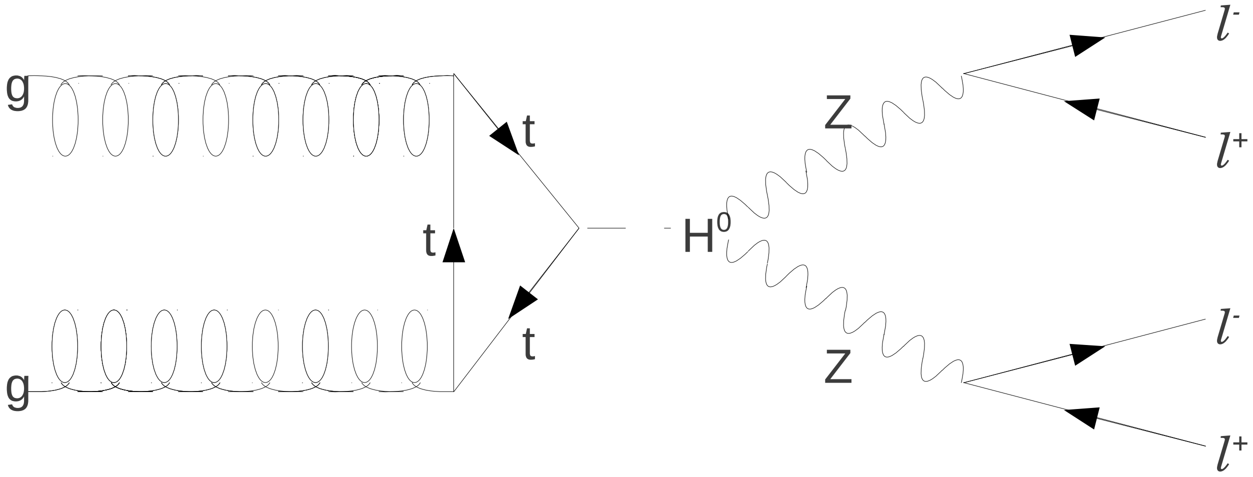

Figure 3.1: Feynman diagram showing the ggF production and decay of a Higgs boson into two Z bosons, which subsequently decay into two leptons each.

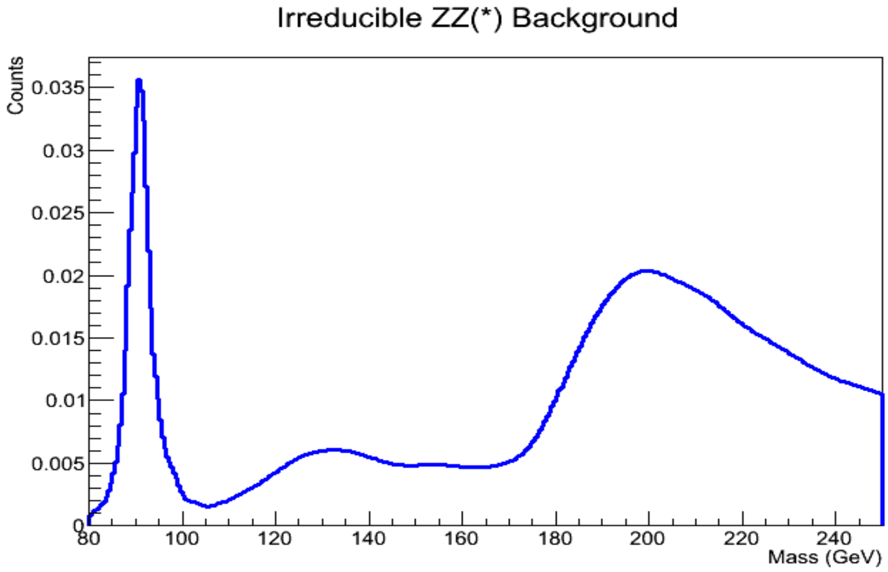

Figure 3.2: Line shape of the ZZ(*) irreducible background continuum.

4. Monte-Carlo Samples

The H → 4l signal peak is simulated using the POWHEG MC event generator which calculates the ggF process with next-to-leading order (NLO) matrix elements [1]. The ggF signal cross section has been computed at up to next-to-next-to leading order (NNLO) in QCD, with NLO electroweak corrections applied, as well as next-to-next-to-leading logarithm QCD soft-gluon re-summations [1]. The MC event reconstruction is provided using a full ATLAS detector simulation with GEANT4 which provides data samples similar as to what the actual physics data would look like. The irreducible ZZ(*) background continuum is modeled using PYTHIA [1].

5. Modeling the Signal Peak

The Higgs mass resonance peak was described using the non-relativistic Breit-Wigner (BW) distribution [5],

$$f_{BW}(x; x^*_{BW}, \sigma_{BW}) = \frac{1}{(x - x^*_{BW})^2 + \frac{\sigma^2_{BW}}{4}} \quad (1)$$

This distribution is completely described by a location parameter, the so called mean, x^*_{BW}, which defines the position of the distribution's maximum, and the width parameter, σ_{BW}, which is the full-width-at-half-maximum. The non-relativistic form was chosen as the mass of the Higgs is large enough that relativistic corrections are negligible at the energy scales involved.

Detector effects – such as radiative losses, and detector resolution – will distort the BW shape. For this reason the signal model is convoluted with a detector response modeled using the Crystal-Ball function [6],

$$f_{CB}(x; x^*_{CB}, \sigma_{CB}, \alpha, n) = \begin{cases} exp\left[-\frac{(x-x^*_{CB})^2}{2\sigma^2_{CB}}\right], & \frac{x-x^*_{CB}}{\sigma_{CB}} > |\alpha| \\ A(B - \frac{x-x^*_{CB}}{\sigma_{CB}})^{-n}, & \frac{x-x^*_{CB}}{\sigma_{CB}} \leq |\alpha| \end{cases}$$
$$A = \left(\frac{n}{|\alpha|}\right)^n \cdot exp\left(-\frac{|\alpha|^2}{2}\right) \quad (2)$$
$$B = \frac{n}{|\alpha|} - |\alpha|$$

Derived by the Crystal Ball collaboration at SLAC, f_{CB} is composed of a central Gaussian with mean and sigma x^*_{CB} and σ_{CB} respectively. This is modified by a power law, low-energy tail of power n joined to the Gaussian at the point $x^*_{CB} - \sigma_{CB}\,\alpha$ [6]. The model used to describe the signal is then given by the convolution of f_{BW} and f_{CB},

$$f_{sig} = f_{BW} \bigotimes f_{CB} \quad (3)$$

Using this model one can fit the mass signal and then deconvolve the result to get a clear separation between the detector and physics contributions to the signal.

Two separate methods can be used to determine the best model to fit the signal peak. The first – believed to be the more physical – keeps the width parameter σ_{BW} of f_{BW} fixed to the theoretical natural width of the Higgs. In this way the detector resolution can be interpreted entirely by the sigma of f_{CB} . The other method makes no assumptions on the natural width of the Higgs and allows σ_{BW} to float in the fits. For both cases the BW mean is used to fit for the Higgs mass. In this analysis, only the case where the width was allowed to float is presented as it gave the better end result.

6. Analysis

The procedure for building the final signal model was carried out in three general steps. First the MC samples were fit using the model described by eqtn. 3 with all model parameters floating except the BW mean, which was fixed to the truth mass of the Higgs. Next, the model parameters were expressed as functions of the BW mean by fitting the values obtained from the first step. Finally, “toyMC” experiments were run with all parameters fixed to the functional forms obtained from the last step, and the pull distributions for the fitted BW mean were produced. If the model is unbiased, with well estimated uncertainties on the fitted mass, it should produce a pull distribution with a standard Gaussian shape [7].

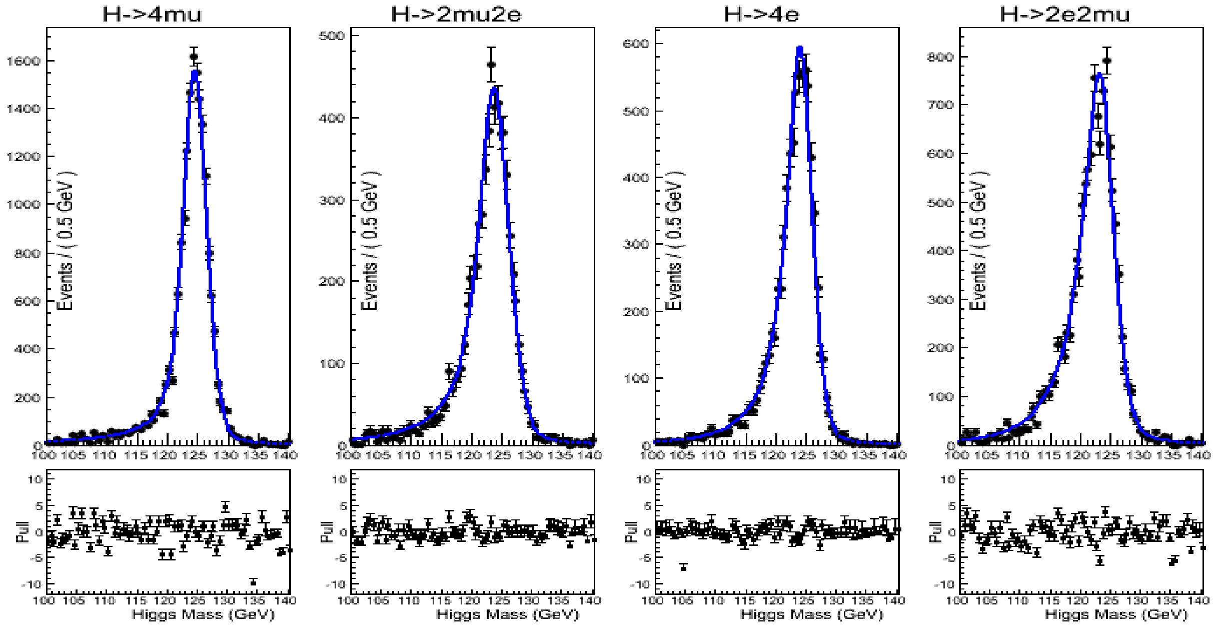

Figure 6.1: Fits and normalized residuals (pulls) of fits for the reconstructed Higgs mass signal at a mass of 125 GeV for the four H->4l channels. The width parameter of the BW function was allowed to float for the fits.

In the first step the MC samples were first divided into four separate subsets corresponding to the four decay channels available in the H->4l decay. These samples were then fit for Higgs' masses in the range from 110 – 200 GeV in 5 GeV steps providing a total of 19 fits. The fitting ranges and initial guesses at the fitted parameters were tuned so as to produce stable fits that converged for all masses used. The best fit values and uncertainties for each parameter were stored. The relative fraction of events in each of the four decay channels were also stored as this was needed later in the analysis. Figure 6.1 shows the fits for each of the decay channels at a Higgs mass of 125 GeV. The normalized residuals, or pulls, of the fits, defined as the difference between the model and data divided by the uncertainty, are included underneath each of the plots to show how closely the model matched the data.

To parameterize the model each individual parameter was plotted and fitted. The parameters of the CB function x^*_{CB}, and σ_{CB} were fit with either a 1st or 2nd degree polynomial depending on which minimized the χ^2/ndf. The two tail parameters α and n of the CB function were fit with either a constant or 1st degree polynomial again depending on which minimized the χ^2/ndf. Finally, the BW width parameter was fit with either a 2nd or 3rd degree polynomial. In all cases the relative fraction of events in each of the decay channels were fit with a 2nd degree polynomial. The coefficients determined from the fit for each parameter are summarized in Table 6.1 and the actual fits for the 4mu channel are shown in Figure 6.2.

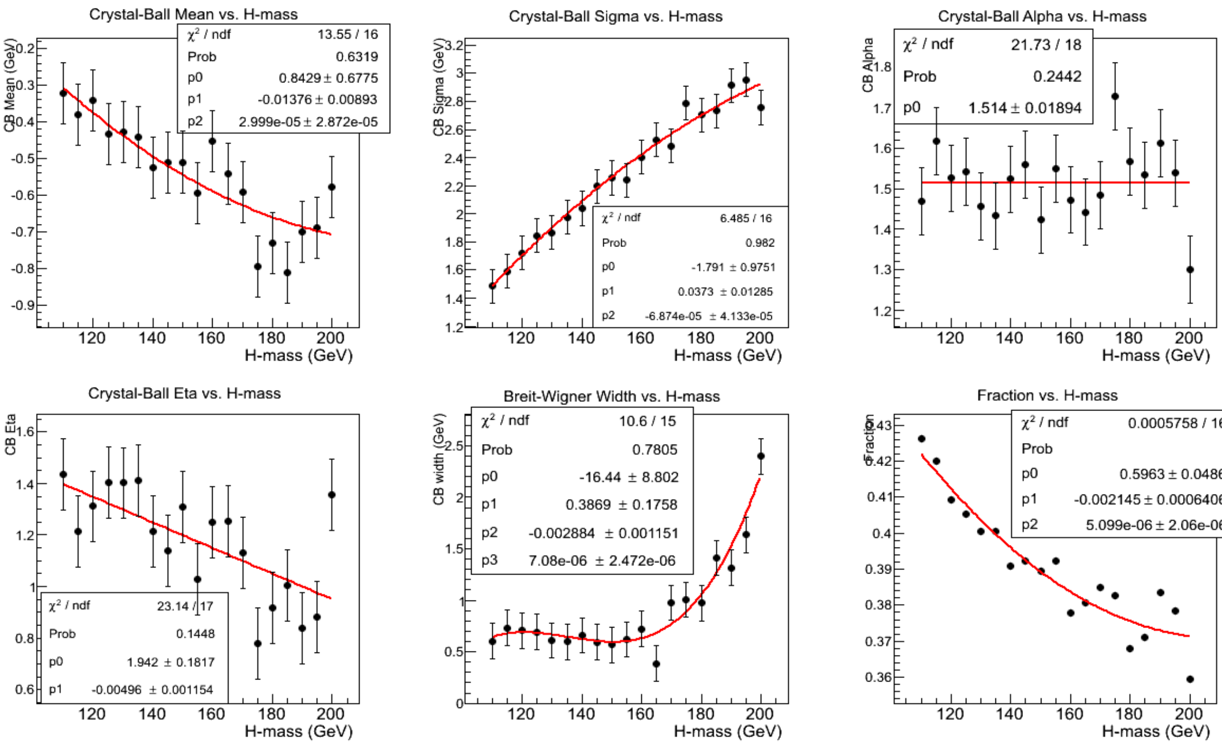

Figure 6.2: Fits to the four CB parameters, BW width, and fraction of events in the 4mu decay channel. The data points correspond to the best fit values taken from the fits to the MC Higgs mass peak samples. The error bars for the 5 model parameters are statistical only. The fraction of events are without errors.

Table 6.1: Polynomial coefficients determined from fits to the different model parameters for the 4mu decay channel. The coefficients for the other channels are listed in Appendix A.

Parameter	p0	p1	p2	p3	χ^2 / ndf
x^*_{CB}	0.84 ± 0.68	-0.014 ± 0.009	(3.0 ± 2.9)e-5	–	13.55 / 16
σ_{CB}	-1.79 ± 0.98	0.037 ± 0.013	(-6.9 ± 4.1)e-5	–	6.485 / 16
α	1.514 ± 0.019	–	–	–	21.73 / 18
n	1.94 ± 0.18	-0.0050 ± 0.0012	–	–	23.14 / 17
σ_{BW}	-16.4 ± 8.8	0.39 ± 0.18	-0.0029 ± 0.0012	(7.1 ± 2.5)e-6	10.6 / 15
fraction	0.596 ± 0.049	-0.00215 ± 0.00064	(5.1 ± 2.1)e-6	–	–

The final step in this procedure was to validate the parameterization developed in the previous steps by generating and analyzing a large number of toyMC experiments. The validation was only completed for a Higgs mass of 125 GeV. The validation was carried out in a number of steps: first, individual toy MC's were run for each of the four decay channels with and without the ZZ(*) background included. Secondly, the four decay channels were combined and the toy MC's were run again using a simultaneous fit to all of the decay channels. Finally, this simultaneous fit was repeated with the ZZ(*) background included.

For the individual fits a data sample was generated with 20 random events from a pdf created using the POWHEG MC data. The sample of twenty events was chosen as this will be the number of Higgs events expected in the ATLAS data for the upcoming year. The model was then fit to the sample 10,000 times, and the best fit value and uncertainty on x^*_{BW} was stored for each fit. The distribution of the pulls and residuals for x^*_{BW} were then plotted and fit with a Gaussian. This procedure was repeated for data samples where the ZZ(*) background was included. The expected number of events in the background (n_{bkg}) spectrum was set to 250, and the expected number of events for the signal (n_{sig}) was set to ten. Both of n_{bkg} and n_{sig} were not fixed at these values as they were modified by an additional Poisson term so that each was effectively the mean of a Poisson distribution. The model was then fit to the data and the pull and residual distributions were analyzed. The pull and residual distributions are shown in Figure 6.3 for the 4mu decay channel with and without background. The same plots for the remaining channels are shown in Appendix B. The Gaussian means and sigmas from the fits to the pull and residual distributions for each of the four decay channels are summarized in Table 6.2 for data without background, and Table 6.3 for data with the ZZ(*) background included.

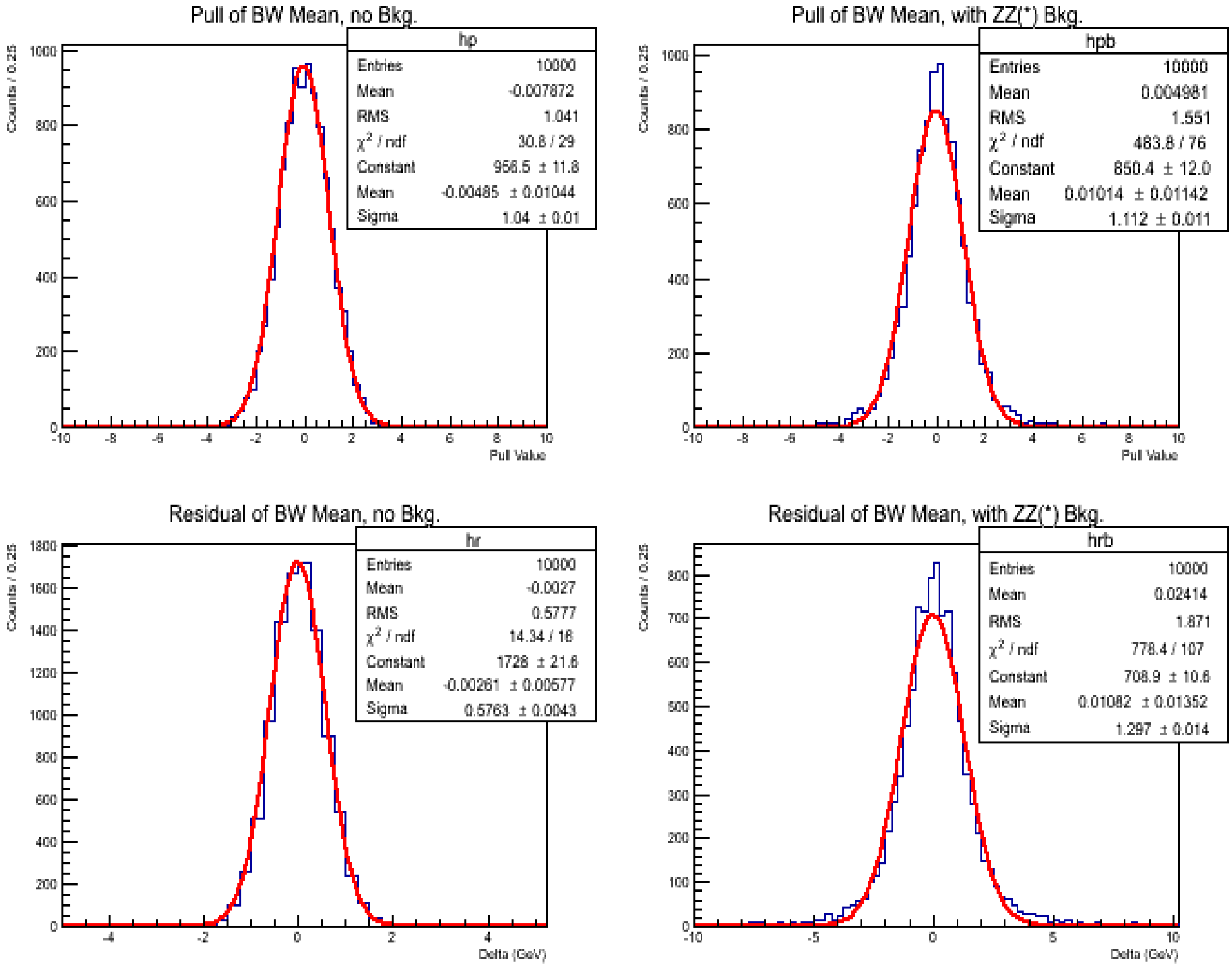

Figure 6.3: Pull, and residual distributions of x^*_{BW} for the 4mu channel from data samples with and without background.

Table 6.2: Gaussian means and sigmas from the fits to the pull and residual distributions from the toy MC validation without background for each of the four decay channels.

Decay Channel	Pull Distribution		Residual Distribution	
	Mean	Sigma	Mean	Sigma
4mu	-0.005 ± 0.010	1.04 ± 0.01	-0.0026 ± 0.0058	0.576 ± 0.004
2mu2e	-0.048 ± 0.011	1.057 ± 0.008	-0.0425 ± 0.0078	0.768 ± 0.006
4e	0.078 ± 0.010	1.016 ± 0.007	-0.0437 ± 0.0065	0.643 ± 0.005
2e2mu	0.0207 ± 0.0099	0.979 ± 0.007	0.0101 ± 0.0080	0.789 ± 0.006

Table 6.3: Gaussian means and sigmas from the fits to the pull and residual distributions from the toy MC validation with the ZZ(*) background for each of the four decay channels.

Decay Channel	Pull Distribution		Residual Distribution	
	Mean	Sigma	Mean	Sigma
4mu	0.0101 ± 0.011	1.11 ± 0.01	0.0108 ± 0.014	1.30 ± 0.01
2mu2e	-0.011 ± 0.013	1.25 ± 0.01	-0.042 ± 0.021	2.02 ± 0.02
4e	0.055 ± 0.012	1.13 ± 0.01	0.062 ± 0.016	1.52 ± 0.02
2e2mu	0.030 ± 0.011	1.12 ± 0.01	0.037 ± 0.021	2.00 ± 0.02

For the simultaneous fits a total of 20 events were generated split between the four channels. The relative fraction of events for each of the four decay channels were fixed to the values obtained by evaluating the event fractions at a mass of 125 GeV. Again 10,000 experiments were generated with and without the ZZ(*) background, and Gaussians were fit to the pull and residual distributions. The results of these fits are summarized in Table 6.5, and the actual fits are shown in Figure 6.4.

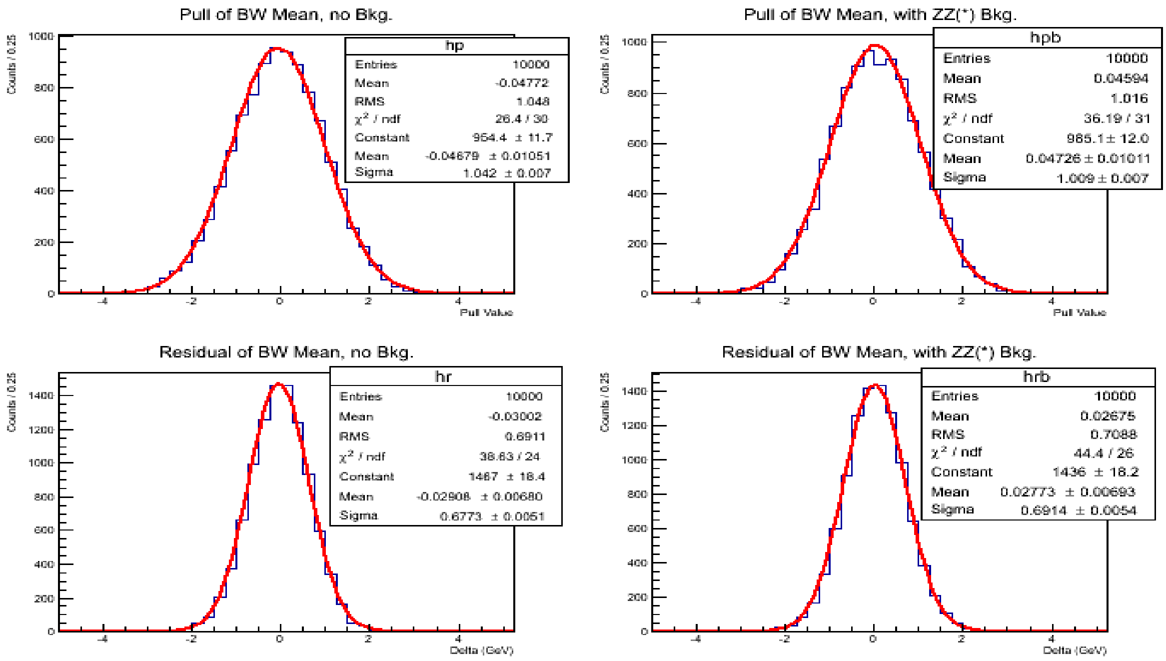

Figure 6.4: Pull and residual distributions for the simultaneous fits to all of the H->4*l* channels with and without the ZZ(*) background included.

Table 6.5: Gaussian means and sigmas from the fits to the pull and residual distributions from the simultaneous toyMC validation with and without the ZZ(*) background.

	Pull Distribution		Residual Distribution	
	Mean	Sigma	Mean	Sigma
No bkg.	-0.046 ± 0.011	1.042 ± 0.007	-0.0291 ± 0.0068	0.6773 ± 0.0061
With bkg.	0.047 ± 0.010	1.009 ± 0.007	0.0277 ± 0.0069	0.6914 ± 0.0054

8. Discussion and Conclusions

The toyMC results for each of the individual fits for the four decay channels without background gave a pull distribution very close to that of a standard Gaussian, though some discrepancies were present. The fit to the pull distribution for the 4mu channel had a mean and sigma of -0.005 ± 0.010 and 1.04 ± 0.01 respectively showing that the model was unbiased and provided a good estimate of the uncertainties on the mass measurement. Similar results were found for the rest of the decay channels as well. For the toyMC experiments with the ZZ(*) background included, the pull distributions did not turn out as nicely. As well as being biased and over or underestimating the uncertainties, all channels had shapes uncharacteristic of Gaussian functions as shown by the large tails and peaks of the distributions.

The results of the toyMC experiments for the simultaneous fit of all the four decay channels were similar to the individual channels when the ZZ(*) background was not included. In this case the fit to the pull distribution gave a mean and sigma of -0.046 ± 0.011 and 1.042 ± 0.007 respectively. For the case where the background was included much better results were obtained then in the individual case. In this case there was a small right-bias with well estimated uncertainties as the mean and sigma were found to be 0.047 ± 0.010 and 1.009 ± 0.007 respectively.

It is somewhat curious as to why the toyMC experiments for the individual channels with background produced such bad results compared with the case with the simultaneous fit of all channels. A possible reason for this observation may be due to a differing fraction of physics-events to background-events between the individual and simultaneous fits. For the individual channels each toyMC experiment was generated with 10 physics events and 250 background events. For the simultaneous toyMC a total of 20 physics events were generated split between the four channels. This difference may have caused the fits to be more stable for the simultaneous case resulting in more reliable values of the mean and better estimates of the fitted mass uncertainty. An easy check to see if this was the case could be done by repeating the individual toyMC experiments using the same physics-event to background-event fraction for each of the four decay channels as was used in the simultaneous case.

In the cases where the sigmas of the pull distributions were near one, we can look at the residual distributions to obtain an estimate on the uncertainty of a Higgs mass measurement. For the four different channels the sigmas of the fitted Gaussians were determined to be ~ 0.6 GeV without background. For the simultaneous fits the sigma values were ~ 0.7 GeV with and without the background included. The most recent ATLAS paper quotes a value for the Higgs mass of 126.0 ± 0.4 (stat) ± 0.4 (sys) GeV [1], which makes an uncertainty of 0.7 GeV at least comparable.

From the results and discussion presented above the model developed appears to be a good candidate to use for a measurement of the Higgs mass. Although the pull distributions for the individual-channel toyMC experiments with background did not agree with what was expected, the simultaneous fit with the ZZ(*) background did give very reasonable results. As an actual measurement of the Higgs mass will be made by fitting all decay channels together, the results of the simultaneous fit provide the best evidence for this model. Whether or not this model can give a more precise measurement of the Higgs mass then the current quoted value is not yet known.

However, if we consider the value of 0.7 GeV as a good estimate for this uncertainty, it is definitely possible that, after some fine tuning, a more precise measurement could be made.

9. Future Work

Many tasks still need to be completed in order to better determine the efficacy of this model. One of the main, and most obvious, tasks is to improve the parameterization to produce a truly unbiased model that still gives a good estimate to the uncertainty on a mass measurement. It is believed that this can be accomplished with more high-statistics MC samples in a smaller window around a mass of 125 GeV. The model should also be tested with more than just the irreducible ZZ(*) background included as this will not be the case for an actual mass measurement. It would also be desirable to split up the physics and detector effects completely between the two convoluted functions used in the model. To do this with the model tested in this paper necessitates fixing the BW width to the theoretical natural width of the Higgs, though, as mentioned previously, doing this did not produce good results. Instead a different function may be used instead of the BW that itself takes care of radiation losses, while still allowing the CB to model the detector effects.

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Appendix A – Parameterization for the H → 2mu2e, 4e, and 2e2mu Channels

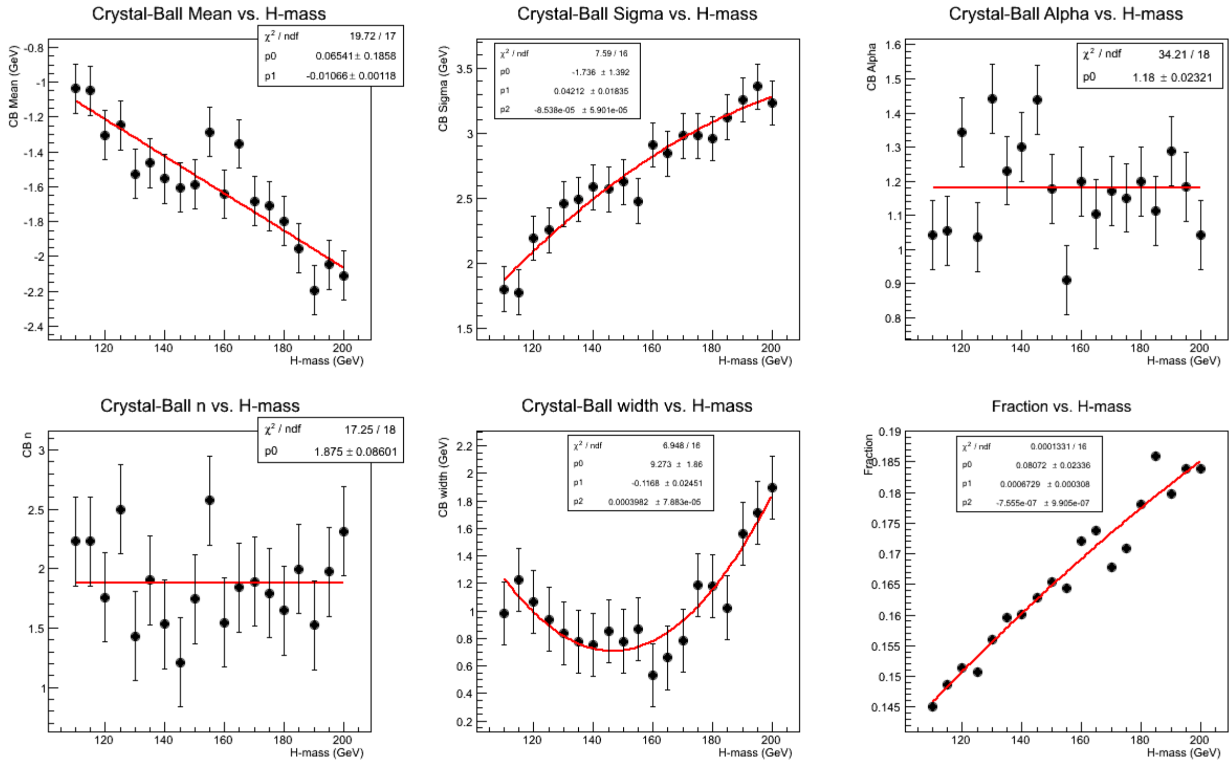


Figure A.1: Fits to the four CB parameters, BW width, and fraction of events in the 2mu2e decay channel. The data points correspond to the best fit values taken from the fits to the MC Higgs mass peak samples. The error bars for the 5 model parameters are statistical only. The fraction of events are without errors.

Table A.1: Polynomial coefficients determined from fits to the different model parameters for the 2mu2e decay channel.

Parameter	p0	p1	p2	p3	χ^2 / ndf
x^*_{CB}	0.07 ± 0.19	-0.0107 ± 0.0012	-	-	19.72 / 17
σ_{CB}	0.042 ± 0.018	(-8.5 ± 5.9)e-5	-	-	7.59 / 16
α	1.180 ± 0.023	-	-	-	34.21 / 18
n	1.875 ± 0.086	-	-	-	17.25 / 18
σ_{BW}	9.3 ± 1.9	-0.117 ± 0.025	(3.98 ± 0.79)e-4	-	6.948 / 16
fraction	0.081 ± 0.023	(6.7 ± 3.1)e-4	(-7.6 ± 9.9)e-7	-	-

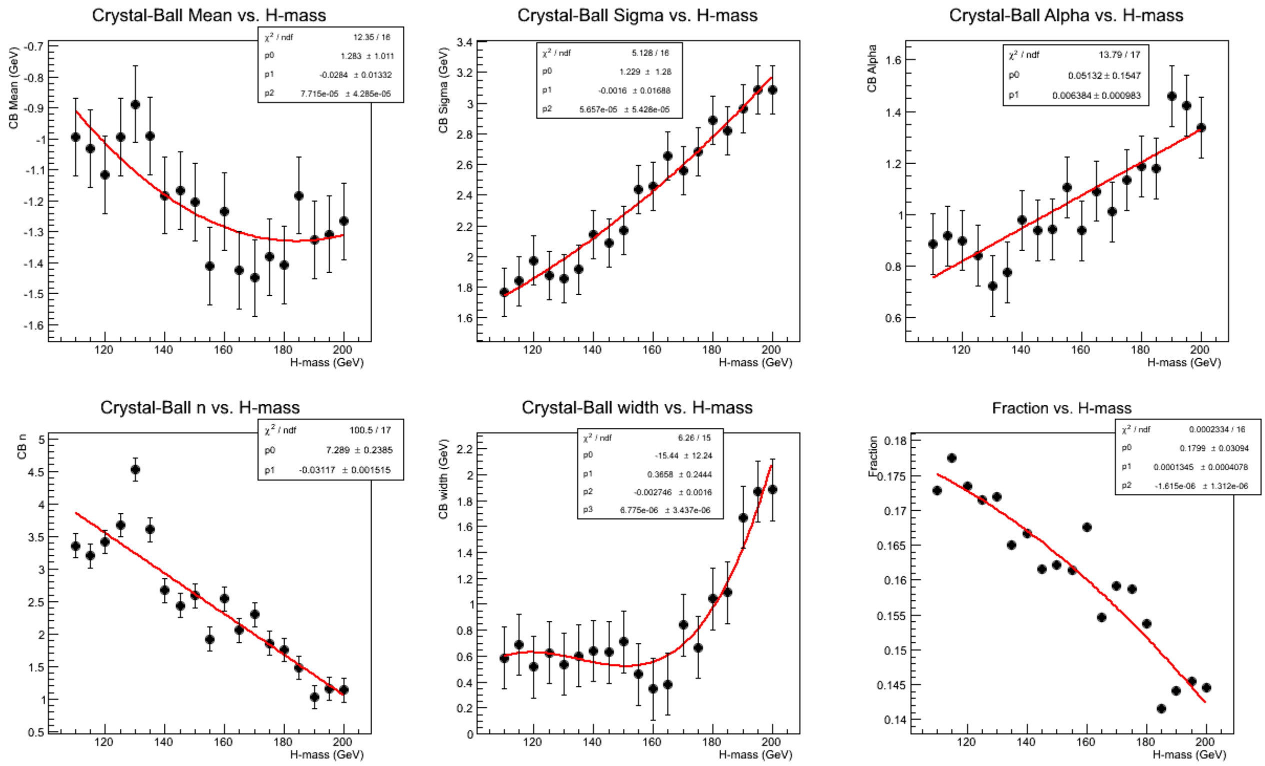
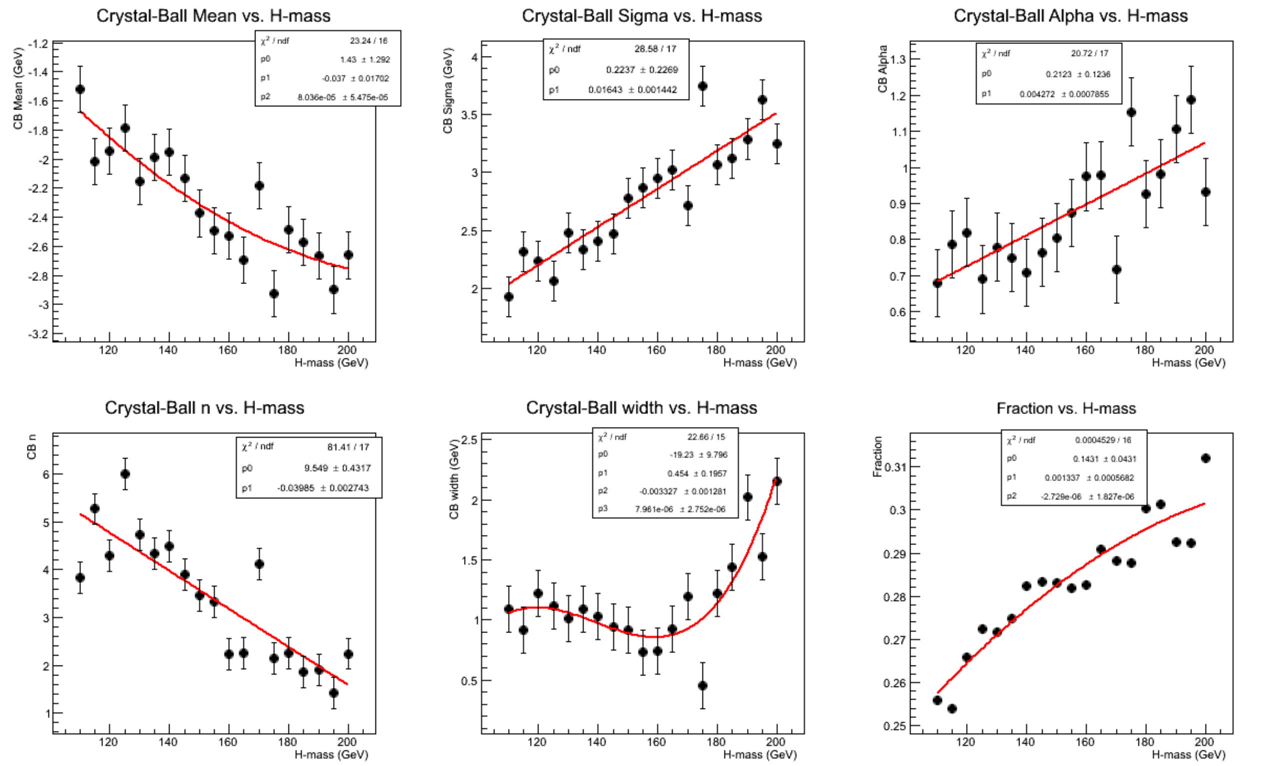


Figure A.2: Fits to the four CB parameters, BW width, and fraction of events in the 4e decay channel. The data points correspond to the best fit values taken from the fits to the MC Higgs mass peak samples. The error bars for the 5 model parameters are statistical only. The fraction of events are without errors.

Table A.2: Polynomial coefficients determined from fits to the different model parameters for the 4e decay channel.

Parameter	p0	p1	p2	p3	χ^2 / ndf
x^*_{CB}	1.3 ± 1.0	-0.028 ± 0.013	(7.7 ± 4.3)e-5	-	12.35 / 16
σ_{CB}	1.23 ± 1.28	-0.002 ± 0.016	(5.7 ± 5.4)e-5	-	5.128 / 16
α	0.05 ± 0.15	0.00638 ± 0.00098	-	-	13.79 / 17
n	7.29 ± 0.24	-0.0312 ± 0.0015	-	-	100.5 / 17
σ_{BW}	-15 ± 12	0.37 ± 0.24	-0.0027 ± 0.0016	(6.8 ± 3.4)e-6	6.26 / 15
fraction	0.180 ± 0.031	(1.3 ± 4.1)e-4	(-1.6 ± 1.3)e-6	-	-



Appendix B – Plots of Pull and Residual Distributions for the Individual H → 2mu2e, 4e, and

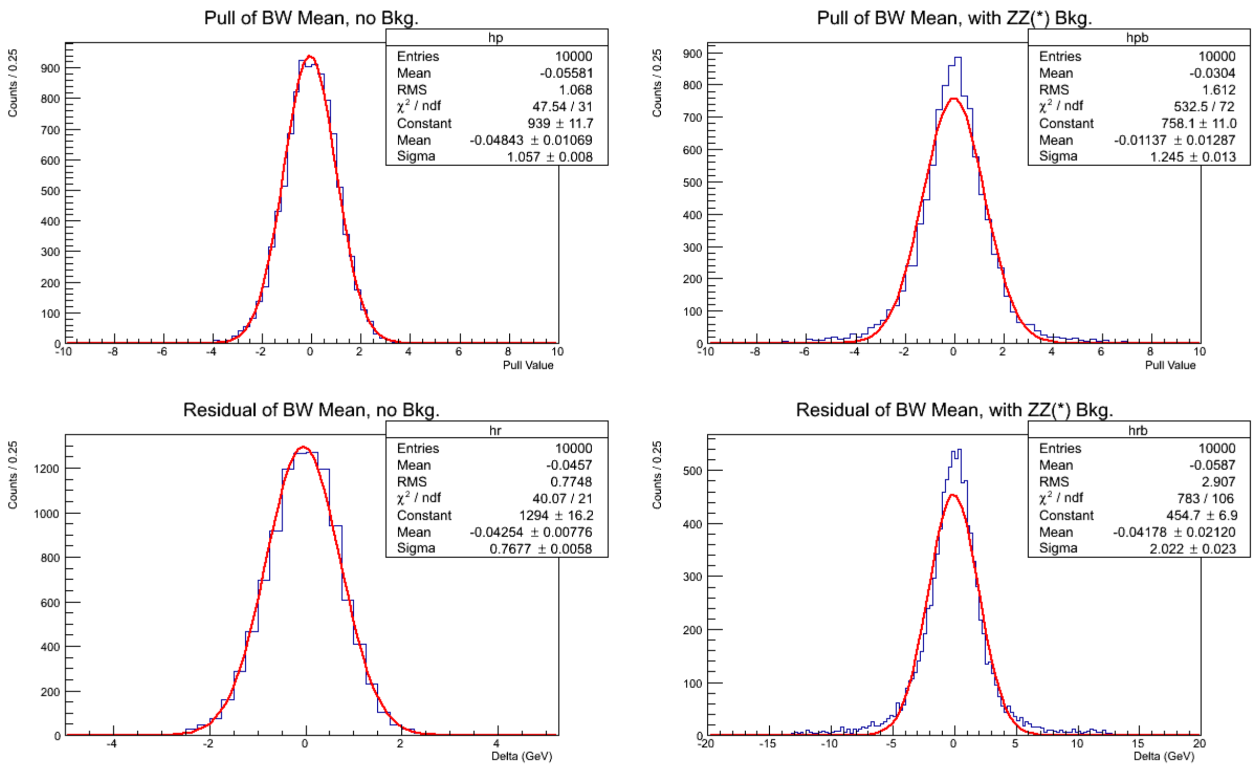


Figure B.1: Pull, and residual distributions of x^*_{BW} for the 2mu2e channel from data samples with and without background.

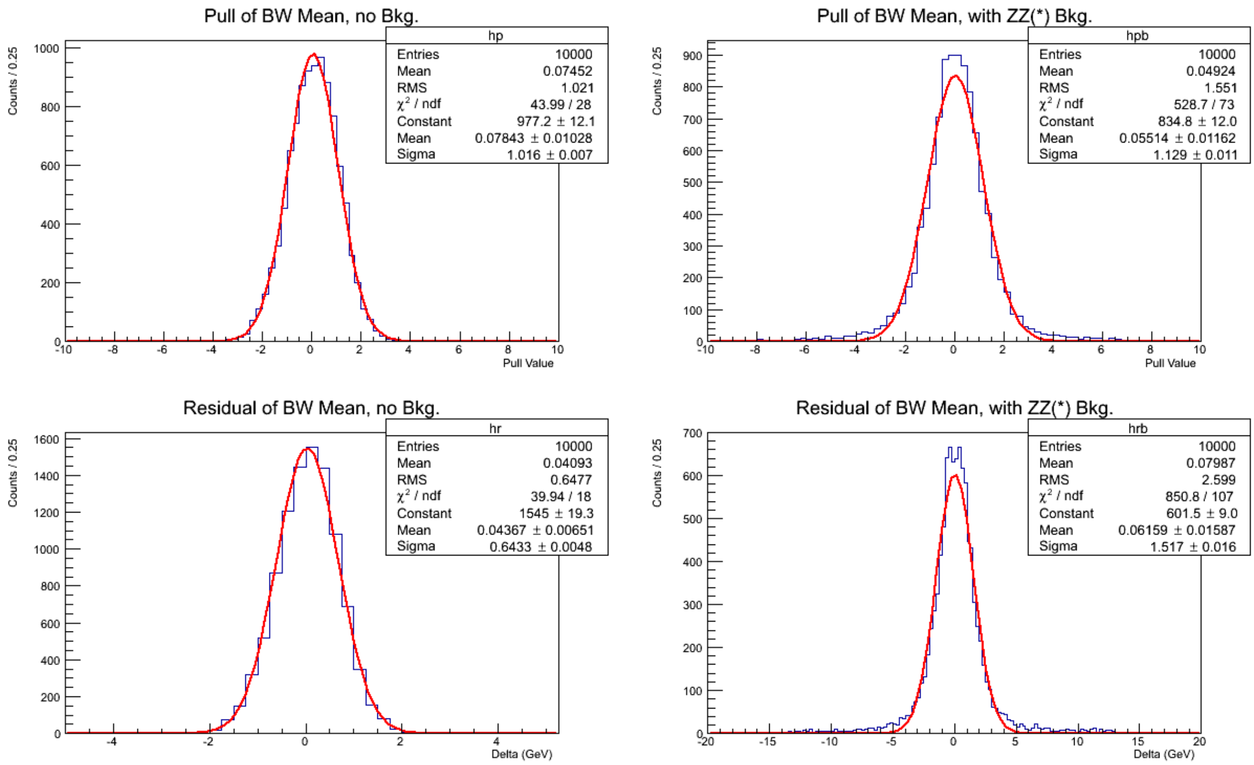


Figure B.2: Pull, and residual distributions of x^*_{BW} for the 4e channel from data samples with and without background.

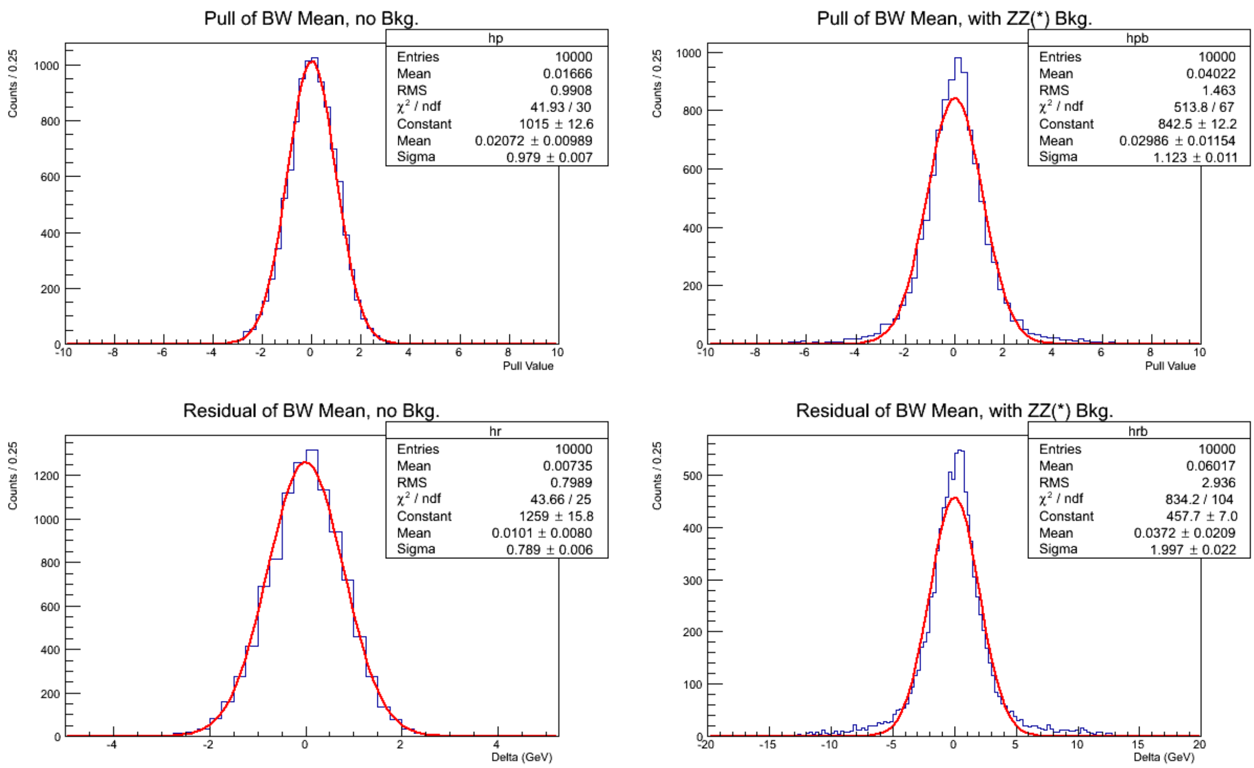


Figure B.3: Pull, and residual distributions of x^*_{BW} for the 4e channel from data samples with and without background.